数学Ⅰ　第11回　関数の微分法

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問題1

(1-1) 積の微分公式 $\{f(x)g(x)\}' = f'(x)g(x) + f(x)g'(x)$ を示せ。

(1-2) 商の微分公式 $\left\{\dfrac{f(x)}{g(x)}\right\}' = \dfrac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$ を示せ。

(1-3) 対数微分の公式 $f'(x) = f(x)\{\log f(x)\}'$ を示せ。

問題2 以下で与えられた関数 y を x で微分し，y' を求めよ．ただし，a, b は定数とする．

(2-1) $y = (x^2+1)^5(3x+1)^4$

(2-2) $y = e^x$

(2-3) $y = \log x$

(2-4) $y = \sin x$

(2-5) $y = \cos x$

(2-6) $y = x + \sqrt{x^2+a}$

(2-7) $y = e^{ax}$

(2-8) $y = e^{-x^2}$

(2-9) $y = \tan x$

(2-10) $y = \sin ax \cos bx$

(2-11) $y = \sin^2 ax$

(2-12) $y = \sin^{-1} x$

(2-13) $y = \cos^{-1} x$

(2-14) $y = \tan^{-1} x$

(2-15) $y = (1+x^2)e^{-x}$

(2-16) $y = e^{\sin x}$

(2-17) $y = x^x$

(2-18) $y = x^{\sin x}$

(2-19) $y = \sqrt{\sin 2x}$

(2-20) $y = \dfrac{x^2-1}{2}\log(1-x) - \dfrac{x}{2} - \dfrac{x^2}{4}$

(2-21) $y = e^{ax}\cos bx$

(2-22) $y = (\log x)^3$

(2-23) $y = \dfrac{1}{\sqrt{\log x}}$

(2-24) $y = \sqrt{\dfrac{1-\cos 2x}{1+\cos 2x}}$

(2-25) $y = e^{ax}\dfrac{a\cos bx + b\sin bx}{a^2+b^2}$

(2-26) $y = \sin x^3$

(2-27) $y = e^{\sin^2 ax}$

(2-28) $y = x\log x - x$

(2-29) $y = \dfrac{\sin 2x}{\sqrt{1-x^2}}$

(2-30) $y = x\cos^2 x$

(2-31) $y = \sin^2 x^3$

(2-32) $y = x^{\sqrt{x}}$

(2-33) $y = \sqrt[3]{\dfrac{x-1}{x+1}}$

(2-34) $y = \log\left|\tan\dfrac{x}{2}\right|$

(2-35) $y = \dfrac{1}{2a}\log\left|\dfrac{x-a}{x+a}\right|$

(2-36) $y = \dfrac{1}{2}(x\sqrt{x^2+a} + a\log|x+\sqrt{x^2+a}|)$

(2-37) $y = \log\left(\dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$

(2-38) $y = \log\left|x+\sqrt{x^2+1}\right|$

(2-39) $y = \dfrac{1}{3}\tan^3 x + 3\tan x - \dfrac{3}{\tan x} - \dfrac{1}{3\tan^3 x}$

(2-40) $y = \dfrac{1}{2}\log|\cos x + \sin x| + \dfrac{x}{2}$

(2-41) $y = x\sin^{-1} x + \sqrt{1-x^2}$

(2-42) $y = 2a\tan^{-1}\sqrt{\dfrac{a+x}{a-x}} - \sqrt{a^2-x^2}$

(2-43) $y = \dfrac{a^2}{2}\sin^{-1}\dfrac{x}{a} - \dfrac{1}{2}x\sqrt{a^2-x^2}$

(2-44) $y = \dfrac{1}{\sqrt{3}}\log\left|\dfrac{\sqrt{3}+\tan\frac{x}{2}}{\sqrt{3}-\tan\frac{x}{2}}\right|$

(2-45) $y = \dfrac{1}{4\sqrt{2}}\log\left|\dfrac{x^2+\sqrt{2}x+1}{x^2-\sqrt{2}x+1}\right| + \dfrac{1}{2\sqrt{2}}\left(\tan^{-1}(\sqrt{2}x+1) + \tan^{-1}(\sqrt{2}x-1)\right)$