

問題 1

(1-1) 積の微分公式 $\{f(x)g(x)\}' = f'(x)g(x) + f(x)g'(x)$ を示せ。

(1-2) 商の微分公式 $\left\{\frac{f(x)}{g(x)}\right\}' = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$ を示せ。

問題 2

以下で与えられた関数 y を x で微分し、 y' を求めよ。ただし、 a, b, c, \dots は定数とする。

(2-1) $y = (x^2 + 1)^5(3x + 1)^4$

(2-21) $y = e^{\sin^2 ax}$

(2-2) $y = x + \sqrt{x^2 + a}$

(2-22) $y = x \log x - x$

(2-3) $y = e^{ax}$

(2-23) $y = \log \left| \tan \frac{x}{2} \right|$

(2-4) $y = e^{-x^2}$

(2-24) $y = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right|$

(2-5) $y = \tan x$

(2-25) $y = \frac{1}{2}(x\sqrt{x^2+a} + a \log |x + \sqrt{x^2+a}|)$

(2-6) $y = \sin ax \cos bx$

(2-26) $y = \log \left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \right)$

(2-7) $y = \sin^2 ax$

(2-27) $y = \log |x + \sqrt{x^2+1}|$

(2-8) $y = \sin^{-1} x$

(2-28) $y = \frac{1}{3} \tan^3 x + 3 \tan x - \frac{3}{\tan x} - \frac{1}{3 \tan^3 x}$

(2-9) $y = \cos^{-1} x$

(2-29) $y = \frac{1}{2} \log |\cos x + \sin x| + \frac{x}{2}$

(2-10) $y = \tan^{-1} x$

(2-30) $y = x \sin^{-1} x + \sqrt{1-x^2}$

(2-11) $y = (1+x^2)e^{-x}$

(2-31) $y = 2a \tan^{-1} \sqrt{\frac{a+x}{a-x}} - \sqrt{a^2-x^2}$

(2-12) $y = e^{\sin x}$

(2-32) $y = \frac{a^2}{2} \sin^{-1} \frac{x}{a} - \frac{1}{2} x \sqrt{a^2-x^2}$

(2-13) $y = x^x$

(2-33) $y = \frac{1}{\sqrt{3}} \log \left| \frac{\sqrt{3} + \tan \frac{x}{2}}{\sqrt{3} - \tan \frac{x}{2}} \right|$

(2-14) $y = x^{\sin x}$

(2-34) $y = \frac{1}{4\sqrt{2}} \log \left| \frac{x^2 + \sqrt{2}x + 1}{x^2 - \sqrt{2}x + 1} \right| + \frac{1}{2\sqrt{2}} \left(\tan^{-1}(\sqrt{2}x + 1) + \tan^{-1}(\sqrt{2}x - 1) \right)$

(2-15) $y = \sqrt{\sin 2x}$

(2-16) $y = \frac{x^2-1}{2} \log(1-x) - \frac{x}{2} - \frac{x^2}{4}$

(2-17) $y = e^{ax} \cos bx$

(2-18) $y = (\log x)^3$

(2-19) $y = \frac{1}{\sqrt{\log x}}$

(2-20) $y = \sin x^3$