

数学演習I 第10回 関数の微分法

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以下で与えられた関数 y を x で微分し、 y' を求めよ。ただし、 a, b, c, \dots は定数とする。

1. $y = (x^2 + 1)^5(3x + 1)^4$

$$y' = 10x(x^2 + 1)^4(3x + 1)^4 + 12(x^2 + 1)^5(3x + 1)^3$$

2. $y = x + \sqrt{x^2 + a}$

$$y' = 1 + \frac{x}{\sqrt{x^2 + a}}$$

3. $y = e^{ax}$

$$y' = ae^{ax}$$

4. $y = e^{-x^2}$

$$y' = -2xe^{-x^2}$$

5. $y = \tan x$

$$y' = \frac{1}{\cos^2 x}$$

6. $y = \sin ax \cos bx$

$$y' = a \cos ax \cos bx - b \sin ax \sin bx$$

7. $y = \sin^2 ax$

$$y' = a \sin 2ax$$

8. $y = \sin^{-1} x$

$$y' = \frac{1}{\sqrt{1-x^2}}$$

9. $y = \cos^{-1} x$

$$y' = -\frac{1}{\sqrt{1-x^2}}$$

10. $y = \tan^{-1} x$

$$y' = \frac{1}{1+x^2}$$

11. $y = (1+x^2)e^{-x}$

$$y' = -(x-1)^2e^{-x}$$

12. $y = e^{\sin x}$

$$y' = \cos x e^{\sin x}$$

13. $y = x^x$

$$y' = (\log x + 1)x^x$$

14. $y = x^{\sin x}$

$$y' = \left(\cos x \log x + \frac{\sin x}{x} \right) x^{\sin x}$$

$$15. \ y = \sqrt{\sin 2x}$$

$$y' = \frac{\cos 2x}{\sqrt{\sin 2x}}$$

$$16. \ y = \frac{x^2 - 1}{2} \log(1 - x) - \frac{x}{2} - \frac{x^2}{4}$$

$$y' = x \log(1 - x)$$

$$17. \ y = e^{ax} \cos bx$$

$$y' = (a \cos bx - b \sin bx)e^{ax}$$

$$18. \ y = (\log x)^3$$

$$y' = \frac{3}{x}(\log x)^2$$

$$19. \ y = \frac{1}{\sqrt{\log x}}$$

$$y' = -\frac{1}{2x}(\log x)^{-\frac{3}{2}}$$

$$20. \ y = \sin x^3$$

$$y' = 3x^2 \cos x^3$$

$$21. \ y = e^{\sin^2 ax}$$

$$y' = a \sin 2ax e^{\sin^2 ax}$$

$$22. \ y = x \log x - x$$

$$y' = \log x$$

$$23. \ y = \log \left| \tan \frac{x}{2} \right|$$

$$y' = \frac{1}{\sin x}$$

$$24. \ y = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right|$$

$$y' = \frac{1}{x^2 - a^2}$$

$$25. \ y = \frac{1}{2}(x\sqrt{x^2 + a} + a \log |x + \sqrt{x^2 + a}|)$$

$$y' = \sqrt{x^2 + a}$$

$$26. \ y = \log \left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \right)$$

$$y' = \frac{1}{x\sqrt{1-x^2}}$$